# On the Behavior of a Very Fast Bidirectional Bus Network

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*Abstract*—**In this paper we study the behavior of a very fast bidirectional bus system. The bidirectional bus system has been investigated in the past under the main assumption that the propagation delay incurred by a packet is relatively small in comparison to its transmission time. Under this assumption, it has been shown that if the packet transmission time decreases, the performance of existing access schemes (like CSMA) degrades. Recent technological developments (such as fiber optics) in communication networks have made possible much faster bus networks. For these networks it no longer may be assumed that the propagation delay is relatively small in comparison to the transmission time. This paper deals with analyzing the very fast bidirectional bus system. In contrast to previous studies, the assumption that the bus is very fast is inherently embedded in the system model. The results derived in this paper show that due to self-synchronization properties observed in the system at high loads, the system performance is not poor as implied from previous studies.**

## I. Introduction and Previous Work

In local area networks, a channel is shared among many stations which are (relatively) close to each other. One of the common topologies for such a network is the bidirectional bus (e.g., Ethernet) and one of the most popular access schemes for this topology is Carrier Sense Multiple Access (CSMA). In CSMA, a station senses the channel before transmitting; if the channel is idle the station transmits right away, otherwise it stays silent and postpones transmission for a later time. (An improvement of CSMA is CSMA with Collision Detection (CSMA-CD). In this scheme, in addition to carrier sensing, a station can listen to the channel while it is transmitting and therefore can detect if it is involved in a collision. If a collision is detected, the station aborts its transmission and repeats the scheme described above.) Both access schemes take advantage of the very short propagation delay (relative to the transmission time). The ratio between the propagation delay and the packet transmission time is denoted by $a$ and can be thought of as the number of packets "contained in" the bus:

$$a \triangleq \frac{\text{propagation delay}}{\text{packet transmission time}}.$$

The performance of CSMA was studied by Kleinrock and Tobagi in [6], [7], and [16]. The performance of CSMA-CD was studied by Tobagi and Hunt [17] and by Lam [9]. These studies were based on the underlying assumption that the parameter $a$ is small so that packets are heard simultaneously by all stations. Two important properties were observed with respect to these access schemes. 1) The attained *throughput*, $S$, of both systems, increases with the *offered load*, $G$, until it reaches its maximum. After this point (very high load), the throughput decreases. 2) The maximum attainable throughput, denoted traditionally as the *system capacity* (and to be denoted here as the *system efficiency*[1]), decreases with $a$. It is observed that the performance of these schemes is good as long as $a \leq 0.05$ (efficiency of about 70%). For larger values of $a$ (like $a = 1$), the efficiency of these systems may go as low as 20%.

Technological developments (such as fiber optics) in communication networks have recently increased the speed of the communication channel, and future developments are likely to increase it even more. Other technological improvements allow the future networks to be longer and longer. These trends lead the communication industry to the building of systems where the parameter $a$ is larger and larger. One possibility for analyzing these new systems is to follow the approach taken in [6], [7], [16], [17], and [9] and to use the throughput/efficiency expression derived there. Doing so, we soon realize that the efficiency of these systems approaches zero as $a$ increases, and thus the use of CSMA may be very inefficient in these systems.

The goal of this paper is to challenge this "discouraging" result predicting that the throughput of CSMA on very fast networks is very close to zero. We depart from the previous studies by discarding the assumptions that $a$ is small and that packets are instantaneously received by all stations. Instead, we use the fact that $a$ is large as an *underlying assumption* (in fact, we assume that $a = N - 1$, where $N$ is the number of attached stations) and create a model in which the propagation process is *inherently modeled* (rather than being assumed to be instantaneous). The main feature of the model adopted is that different packets are heard at different times by different stations. This creates discrepancy among the stations (rather than uniformity in the previous models) and thus causes the system to have several plausible (and quite surprising) properties. 1) The efficiency of the system, under deterministic and scheduled arrivals, is close to 2 (in contrast to efficiency of 1 in the previous models). 2) Under stochastic arrivals, the system is stable: an increase in the offered load leads to an increase in the throughput. 3) The system efficiency, under stochastic arrivals, and using

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[1] Here we need to elaborate on the terminology. In the traditional literature, throughput is dimensionless and denotes the fraction of time at which successful transmissions are received in the system (usually it is assumed that the transmission of a packet takes one time unit and then throughput is measured in packets per time unit). Capacity, in that terminology, is measured in the same units (i.e., it is dimensionless) and is defined to be the maximal attainable throughput of the system. Here we use a slightly different terminology: we define the *system efficiency* to be identical to what traditionally has been defined as capacity (therefore the units of efficiency are dimensionless) and define *capacity* as the maximal number of bits which can be successfully transmitted (on the average) per time unit in the system; The units of capacity are, therefore, bits per time unit. Note that in our terminology capacity is the product of the transmission rate (measured in bits per time unit) and the efficiency, namely: capacity = transmission rate × efficiency. Note also that the adoption of this terminology (as opposed to the traditional one) is necessary when the analysis involves comparisons of systems with different transmission rates.

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the CSMA efficiency, is 1 (and not close to zero as previously may have been predicted). We thus conclude that due to asymmetry between the stations, the performance of these networks is much better than what would otherwise be predicted by the fully symmetric "traditional" models.

The structure of this paper is as follows. In Section II, the system model is described. In Section III we study the theoretical limitations of the very fast shared bus system. The main goal in that section is to calculate the maximum throughput which can be achieved in the system, neglecting the randomized behavior of the system inputs. Bounds for the system efficiency under several conditions are derived in that section. In Section IV we investigate the system behavior under the assumption of stochastic arrivals. The model used in that section is similar to the models used in the analysis of slotted ALOHA and CSMA; however, in contrast to those models, this model captures the correlation between events occurring in the system. The main property discovered in this analysis is that in contrast to previously studied shared channel systems, this system is very stable and the system throughput increases with the offered load. Concluding remarks are given in Section V.

Last, a word about recent related work. In an independent study, Sohraby, Molle, and Venetsanopoulos [12]–[15] studied the performance of CSMA in fast bus systems. The similarity between the two studies is in the explicit modeling of the packet propagation, and in discovering the network asymmetry which implies good performance. That work is different from ours in some aspects of the modeling and in dealing with systems with *large a* which is bounded to be $a < 1/2$; in contrast, we deal with *very large a* ($a = O(N)$ where $N$ is the number of stations). The behavior of fast bus systems has also been investigated by several other studies. However, those studies concentrated on suggesting semiorganized access schemes for these networks and not on studying the behavior of these networks under the CSMA scheme. The main principle of those schemes is to organize the packets transmitted in the system to efficiently use the channel. These studies are reported in [3]–[5] and [11].

## II. Model Description

The system consists of $N$ stations connected by a bidirectional bus and numbered $1, 2, \cdots, N$ from left to right. It is assumed that the stations are located on the bus such that the distance between every two neighboring stations is exactly one unit. The length of a fixed size packet, measured in terms of distance, is assumed to be smaller than or equal to the distance between two neighboring stations. This implies that the parameter $a$ of this system is $a \geq N - 1$. For simplicity, we assume that the packet size exactly equals the distance between neighboring stations, i.e., $a = N - 1$. Time is slotted with slot size equal to the time required to transmit a packet. The time interval, starting at time $t$ and ending at time $t + 1$, is called the $t$th *slot*. Every packet transmission starts at the beginning of some slot.

Due to these assumptions, it is not sufficient to characterize the system events by their timing only; rather, a time-location characterization of events is required. We therefore represent the system behavior using a time-space domain where the horizontal axis represents the location on the bus and the vertical axis represents time (progressing down the page). The propagation of a packet is represented by a band (see Fig. 1, where station 2 transmits a packet at slot $t$ and stations 1 and 3 hear it at slot $t + 1$).

In contrast to the traditional model, packets which collide are not assumed to destroy each other. Rather, they are assumed to "pass through" each other. For example, consider the two packets depicted in Fig. 1. Packets are concurrently transmitted by stations 2 and 4 at slot $t$. During slot $t + 1$, the packets collide at station 3, and thus neither of them is heard properly by station 3. However, the packets "pass through" each other, so during slot $t + 2$ one of them is heard correctly by station 2 and the other is heard correctly by station 4. This assumption is valid, for example, when the bidirectional channel is implemented by two one-directional fiber buses.

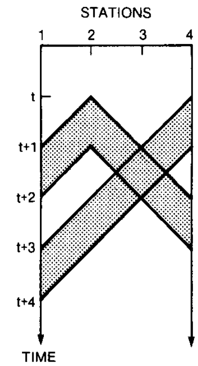


Fig. 1. Two packets pass through each other.

From the above description, it is implied that the terms: *idle slot*, *successful slot*, and *collision slot* are not global properties of the system, but rather local properties of a given station. Therefore, all references to a particular slot will include both time and location reference. For example, we say that in Fig. 1 slot $t + 1$ is an *idle slot* at stations 2 and 4, a *successful slot* at station 1, and a *collision slot* at station 3.

It is important in the context of this model to accurately define the notion of *successful reception*. Although the transmission medium is a *broadcast medium* (i.e., a single packet may be heard by all stations), we assume that the messages themselves are not of the broadcast type but rather of the point-to-point (PTP) type. This means that every packet is destined to a single destination, and only that destination needs to receive it properly. Following this assumption, we define the *successful hearing* and *successful reception* of a packet: a packet is said to be *successfully heard* by station $i$ at slot $t$ if the packet is heard by $i$ at slot $t$, and $t$ is a successful slot at $i$. A packet is said to be *successfully received by station i at slot t* if it is destined to station $i$ and if it is successfully heard by $i$ at slot $t$.

Two important properties of multiple access algorithms are to be discussed in this paper: *politeness* and *fairness*. A station is said to be *polite* if it does not transmit when it hears a transmission originated from another station. Note that politeness is a desired property which is utilized in CSMA algorithms using the Carrier Sense mechanism of the stations. Nevertheless, Carrier Sense in its traditional form will not be very effective in this slotted environment, and needs to be slightly enhanced. This issue can be best explained using Fig. 1 where we consider the action taken by station 2 at slot $t + 2$. In order to prevent station 2 from interfering with the currently passing packet (transmitted from station 4 at slot $t$), it is required that station 2 will be polite at this slot. To enforce this politeness, station 2 needs to make a decision, right at the beginning of slot $t + 2$, whether to transmit or not, based on what appears on the channel at that moment. This can be done only if the station can have some "look-ahead" mechanism, by which it can tell at time $t^-$ what will be the channel status at time $t$. This "look-ahead" mechanism can be easily constructed by bending the bus at the station neighborhood in an $\Omega$ shape, and having the station tapped at the two $\Omega$ legs for "look-ahead" sensing, and to the $\Omega$ head for transmission or reception.

In addition to general politeness, we define directional politeness. A station is said to be *polite to the left (right)* if it does not transmit when it hears a transmission originated at a lower (higher) index station (i.e., a transmission that arrives from the left (right), according to our representation).

A transmission policy is called *fair* if, for every two stations $i$ and $j$, station $j$ is allowed to transmit between any two consecutive transmissions of station $i$. A transmission policy is called *strictly fair* if, for every four stations $i$, $j$, $k$, and $l$, station $i$ is allowed to transmit to station $j$ between any two consecutive transmissions from station $k$ to station $l$.

## III. On the Efficiency of the System

In this section we study the system efficiency under various conditions. The goal is to find the maximum throughput which can be achieved in the system when perfect scheduling is used. The importance in deriving this measure is in understanding the system limitations, and in comparing the system potential to that of other systems. The assumptions used in this section are given in Section II, and the main ones are the following three: 1) the stations are evenly spaced on the bus, 2) the transmissions are slotted and the packet size is equal to the distance between two neighboring stations, and 3) packets "pass through" each other.

To define system throughput, recall that a packet is considered to be successful if it is heard successfully by its destination station. Let $P(t)$ be the number of packets that have been successfully received in the system by time $t$; then the system *throughput*, denoted by $S$, is defined to be $S \triangleq \lim_{t \to \infty} P(t)/t$. The *system efficiency* is defined to be the highest throughput which can be achieved by using a perfect scheduling algorithm (which can perfectly, ahead of time, schedule the transmissions of every station).

For most communication systems, it is straightforward to derive the system efficiency. For example, the efficiency of a system consisting of two stations connected by a point-to-point link is 1, since at most one unit of information can be transmitted on that system per unit of time. Similarly, the efficiency of a fully connected $N$-node network (where each of the links is a point-to-point link and $N$ is even) is $N/2$, since this is the number of conversations that can be concurrently held in the system. The efficiency of the bidirectional bus system, as considered in our paper but under the assumption that the parameter $a$ is small, is 1 since at most one station can successfully transmit at any time.

In contrast to all these systems, the dependency of events in our system both on time and location requires a more careful analysis of the efficiency. In the following we derive both upper and lower bounds on the system efficiency.

### A. Two Upper Bounds on the System Efficiency

Before deriving the bounds, some more notation is required. A point $(i, t)$ in the time-space domain is called a *transmission point* if station $i$ transmits a packet at slot $t$ (i.e., starts transmitting at time $t$). A point $(i, t)$ in the time-space domain is called a *reception point* if station $i$ successfully receives a packet during slot $t$. A line which contains the points $(t, 1)$, $(t+1, 2)$, $(t+2, 3), \cdots, (t+N-1, N)$ is called a *left diagonal* (a diagonal that starts from top left and goes downward and to the right). Similarly, a *right diagonal* is defined. Next, two upper bounds for the system efficiency are derived.

*Theorem 1:* For any scheduling policy, the system throughput obeys: $S \le N/2$.

*Proof:* We assume that there is no transmission prior to $t = 0$. Let $T(t)$ and $R(t)$ be, respectively, the sets of transmission points and reception points $(i, t')$ such that $t' \le t$ $(i = 1, \cdots, N)$. Let $(i, t_1)$ be a reception point in $R(t)$, then there exists a transmission point $(j, t_2) \in T(t)$ which uniquely corresponds to $(i, t_1)$. This is the transmission point which corresponds to the transmission of the packet successfully received at $(i, t_1)$. For this reason, we may conclude that $|T(t)| \ge |R(t)|$. In addition, the two sets $T(t)$ and $R(t)$ must be disjoint since a station cannot transmit and receive concurrently, so the number of points in the joint set cannot exceed the number of points in the $N \times t$ rectangle, namely, $|R(t)| + |T(t)| \le Nt$. Thus, from the two inequalities: $|R(t)|/t \le N/2$, and the claim follows since $|R(t)|$ is the number of packets successfully received by time $t$. □

*Theorem 2:* For any scheduling policy, the system throughput obeys: $S \le 2$.

*Proof:* To prove the claim, consider first a system (called SYS1) consisting of a single *unidirectional bus*. Assuming that the transmission direction is from left to right, we examine the time-space domain and observe that on every left-diagonal there may be at most one reception point. For this reason, the number of reception points in the $N \times t$ rectangle must obey $|R(t)| \le t + N - 1$, and the unidirectional bus throughput is therefore bounded from above by 1.

Next, consider a system (called SYS2) consisting of $N$ stations and two unidirectional buses: one is used to transmit packets from right to left, and the other used to transmit packets on the reverse direction. The activity of a station on one bus may be independent of its activity on the other bus. Thus, for example, a station may transmit on one bus while receiving on the other. Now, it is obvious that the efficiency of SYS2 is bounded by twice the efficiency SYS1. Also, the efficiency of the bidirectional bus system must be bounded from above by the efficiency of SYS2 (simply because the stations in SYS2 are less restricted) and thus the claim follows. □

We therefore conclude that the throughput of any scheduling policy is bounded by:

$$S \le \min(N/2, 2).$$

### B. Achievable Throughput

In this subsection we present lower bounds for the system efficiency under various constraints. First we look at an unconstrained system. It is relatively simple to construct a schedule under which the system throughput is $2 - 2/N$. This schedule is depicted in Fig. 2 where a transmission point is represented by a full dot and a reception point by an empty square. For clarity of the figure, a packet propagation is represented only by a line (representing the "front" part of the packet) and not by a band. Note, however, that this schedule does not obey the fairness restriction since most of the traffic is originated from and destined to the end stations (1 and $N$).

Next we consider fair policies and strictly fair policies. Obviously, the efficiency of these systems is bounded by the efficiency of the unconstrained system. Surprisingly, we find that even the strictly fair system may achieve throughput which is very close to 2. In [10], we constructed a strictly fair policy which achieves throughput of

$$S = 2 - \frac{6N - 8}{N^2 + 2N - 4}.$$

A more efficient transmission pattern for the strictly fair system has been suggested by C. Ferguson. This pattern is depicted in Fig. 3. The throughput attained by this pattern can be calculated by observing that the time to complete the pattern is given by $2 + 3 + \cdots + N = (N+2)(N-1)/2$. The number of packets transmitted in the pattern is $N(N-1)$ (from every station to every other station), and thus the throughput is

$$S = 2 - \frac{4}{N + 2}.$$

While fairness does not impose serious efficiency degradation, politeness does. To understand why politeness decreases efficiency, one should examine Figs. 2 and 3, where the schedules used do not obey the politness restriction. As a matter of fact, it is evident that these schedules "benefit" from letting a station transmit while hearing a packet which is not destined to itself. This degradation is stated in the next theorem.

*Theorem 3:* The efficiency of a polite system is exactly 1.

*Proof:* First we show that the system efficiency is bounded from above by 1. This can be shown by examining the time-space domain and observing that on any left diagonal there can be at most one transmission point, or otherwise the system does not obey politeness. Therefore, the number of transmission points on the $N \times t$ rectangle is bounded by $T(t) \le t + N - 1$. Thus, since the number of packets successfully received by time $t$ is bounded from above by $T(t)$, the system efficiency is bounded from above by 1. Now, it is easy to see that throughput of value 1 is attainable in the system. This can be achieved by having station 1 transmitting all the

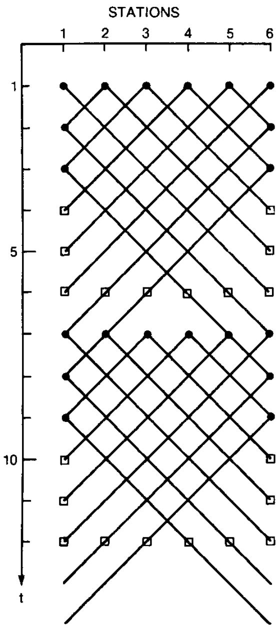



Fig. 2. Throughput of value 10/6 is attainable on a six station system.

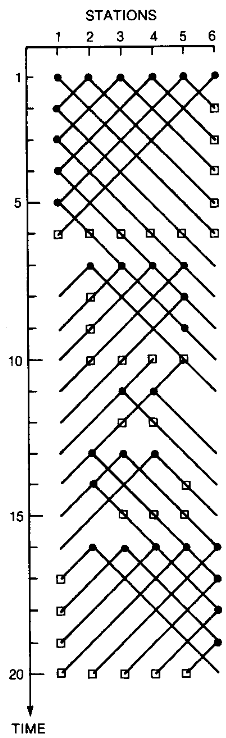



Fig. 3. Strictly fair throughput of value 30/20 is attainable on a six station system.

time and all the other stations stay silent. Thus, we conclude that the efficiency of the system is exactly 1. □

Having calculated the efficiency of a polite system, we next discuss the efficiency of a unidirectional-polite system. We claim that if the direction of politeness can be chosen for every station independently of the politness direction chosen for the other stations, then the efficiency of the system can approach 2. To verify this claim, observe Fig. 2: let station 1 be polite to the left and station 6 be polite to the right (which actually implies no politeness of these stations); let station 2 be polite to the left and station 5 be polite to the right; and let stations 3 and 4 be either polite to the right or polite to the left. Under this politeness rule, the transmission policy depicted in Fig. 2 is still valid and the system throughput can get as high as $2 - 2/N$.

On the other hand, if the politeness direction is chosen to be uniform (i.e., either all stations are polite to the left or all stations are polite to the right), then the system efficiency remains 1. This claim may be easily proved along the same lines of Theorem 3.

### C. Discussion

From this analysis, it is evident that under strict transmission schedules the potential throughput of the fast bidirectional bus system is relatively high. The efficiency of similar single shared-channel systems, like the one-hop packet radio network or the relatively slow bidirectional bus system, is known to be 1. In comparison, we showed above that the time-space event separation observed in the very-fast bus system allows the throughput of this system to get as high as 2. This is shown to hold even if (strict) fairness is required in the system. Thus, these systems can benefit from the use of scheduled transmissions. On the other hand, these results also stress the limitation of the system which cannot accommodate throughput higher than 2.

However, surprisingly, we realize that forcing the politeness property, which is supposed to *increase* the throughput of a bus system under stochastic arrivals (as in the CSMA access scheme), actually *decreases* the system efficiency down to 1. Nevertheless, applying directional politeness does not necessarily degrade the system efficiency.

It should be emphasized that the above results are based on the assumption of point-to-point transmissions. Note that in a broadcast environment, namely, in which each transmission needs to be received by *all* other stations, the capacity is exactly 1 (since each transmission point will require $N - 1$ reception points).

## IV. The System Throughput Under Stochastic Arrivals

The system model is the one given in Section II above. The arrival process is modeled according to the "traditional" model used in the literature (see, for example, [2]) of packet radio networks. According to this model, the packet transmissions of each station are modeled as a sequence of *independent* Bernoulli trials. This sequence represents the combined stream of old retransmitted packets and newly arriving packets. Thus we have:

$$G_i = \Pr[\, i\text{th station transmits a packet in any given slot}] \qquad i = 1, 2, \cdots, N.$$

Since in our model there is importance to the packet destination,[2] we identify the destination of each packet sent:

$$r_{ij} = \Pr[\text{station } i\text{'s packet is destined to station } j] \qquad j \neq i.$$

This definition obviously requires: $\sum_{j \neq i} r_{ij} = 1$ for $i = 1, 2, \cdots, N$.

Two important parameters are considered in this model: the *average traffic* (also called the *offered load*) and the *throughput*. The offered load of station $i$ is the expected number of packets (per slot) transmitted by this station. This is denoted above by $G_i$. Similarly, the offered load from station $i$ to station $j$, denoted by $G_{ij}$, is the expected number of packets transmitted from station $i$ to

[2] The destination information is not important in the traditional model of a slow bus network, since the successful reception of a packet does not depend on its destination.

station $j$. The total offered load of the system, denoted by $G$, is the expected number of packets transmitted (per slot) in the system. Obviously we have $G_{ij} = G_i \cdot r_j$ and $G = \sum_{i=1}^{N} G_i$.

In a similar way, we define the system throughput. The throughput of station $i$, denoted by $S_i$, is the expected number of packets (per slot) originated at station $i$ and successfully received at their destination. Similarly, the throughput from station $i$ to station $j$, denoted by $S_{ij}$, and the total system throughput, denoted by $S$, are defined. Note that this definition of throughput is consistent with the definition given is Section III above.

### A. Exact Throughput Analysis of a Nonpolite System

We start the throughput analysis of the system by studying the nonpolite scheme. In this scheme, the behavior of one station is independent of the transmissions of the other stations; thus, the throughput from station $i$ to station $j$ is easily shown to be

$$S_{ij} = G_i \cdot r_{ij} \cdot \prod_{\substack{k=1 \\ k \neq i}}^{N} (1 - G_k) \qquad i \neq j, \tag{3.1}$$

and the total throughput originated at station $i$ is

$$S_i = \sum_{\substack{j=1 \\ j \neq i}}^{N} S_{ij} = G_i \prod_{\substack{k=1 \\ k \neq i}}^{N} (1 - G_k) \qquad i = 1, 2, \cdots, N. \tag{3.2}$$

It is important to emphasize that the model considered here is significantly different from the one considered by [2]. Nevertheless, the basic assumption that the stations behave independently of each other (by the assumption that no politeness is used) leads both models to the same results. Thus, the throughput of our system is identical to that of the Slotted Aloha system, and we refer the reader to the literature (see, e.g., [8]) for further analysis of its performance.

### B. Polite System: An Exact Analysis

For the analysis of the polite systems, we must change our assumption regarding the arrival process. Rather than using the previous Bernoulli assumption, according to which station $i$ is assumed to transmit a packet with probability $G_i$ at *every slot*, we use a modified assumption according to which station $i$ transmits with probability $G_i$ in each slot in which it is not forced to be silent by the politeness rule. Thus, if we observe only those slots at which station $i$ is allowed to transmit by the politeness rule, the packets transmitted from station $i$ behave like a stream of Bernoulli trials.

Under these assumptions, it is possible to represent the system behavior by a Markov chain. However, note that the stations' status is not sufficient to represent the system. Rather, in order to form a Markov chain, we must include the status of the channel during the $t$th slot in this representation. A state in this Markov chain can be described by the channel status at each of its $N - 1$ segments, where a segment is the channel section between two neighboring stations. During slot $t$ each of these segments may be in either of four states: a) no transmission propagates on the segment, b) transmission from left to right propagates along the segment, c) transmission from right to left propagates along the segment, and d) two concurrent transmissions (from left and from right) propagate along the segment. Since the number of segments is $N - 1$, the state space contains $4^{N-1}$ states.

For very small values of $N$ ($N \leq 5$) it is possible to solve this Markov chain by calculating the (finite) transition matrix and numerically solving for the steady-state probabilities of the system states. To demonstrate the method, consider a two station system. At every slot, the channel may be in either of four states: 1) only station 1 transmits, 2) only station 2 transmits, 3) both stations transmit, and 4) none of the stations transmit. We denote the probability that at slot $t$ the system is in these states by $\pi_1$, $\pi_2$, $\pi_{12}$, and $\pi_0$, respectively. At steady state, these probabilities obey

$$\begin{aligned} \pi_0 &= \pi_0(1 - G_1)(1 - G_2) + \pi_1(1 - G_1) \\ &\quad + \pi_2(1 - G_2) + \pi_{12} \cdot 1 \\ \pi_1 &= \pi_0 G_1(1 - G_2) + \pi_1 G_1 \\ \pi_2 &= \pi_0(1 - G_1)G_2 + \pi_2 G_2 \\ \pi_{12} &= \pi_0 G_1 G_2. \end{aligned}$$

Solving this set of equations yields a solution for $\pi_0$, $\pi_1$, $\pi_2$, and $\pi_{12}$ in terms of the system parameters ($G_1$, $G_2$). From this solution we then get the system throughputs: $S_{12} = \pi_1 + \pi_{12}$, $S_{21} = \pi_2 + \pi_{12}$. Note that the throughput must obey $S_{12} + S_{21} \leq 1$, due to Theorem 3.

To demonstrate the system behavior we next analyze, using this method, a three station system. Two symmetry assumptions are used in this analysis: 1) the transmission rate $G_i$ for symmetrically positioned stations is assumed to be identical (thus, we assume that $G_1 = G_3 = p$ and $G_2 = q$); and 2) the destination of a packet transmitted from station $i$ is equally likely to be any of the other $N - 1$ stations, i.e., $r_{ij} = 1/(N - 1)$ for $j \neq i$ and $r_{ii} = 0$.

Using the method described above, we constructed the Markov chain (consisting of 16 states) representing the system (see [10]), solved it numerically, and calculated the system throughputs. The results of this analysis are depicted in Figs. 4-6. Fig. 4 is a three-dimensional plot of the throughput originated at a side node

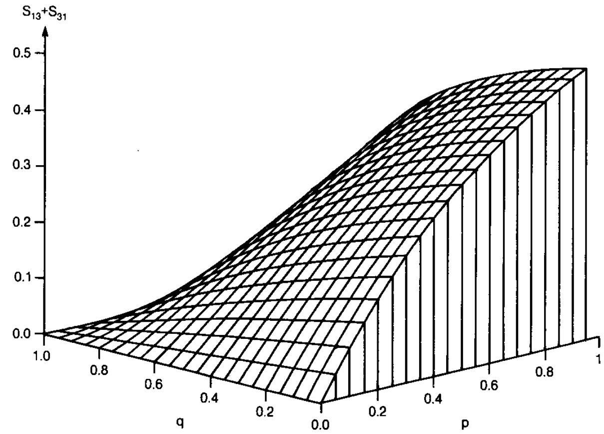



Fig. 4. The side-to-side throughput in a three station system.

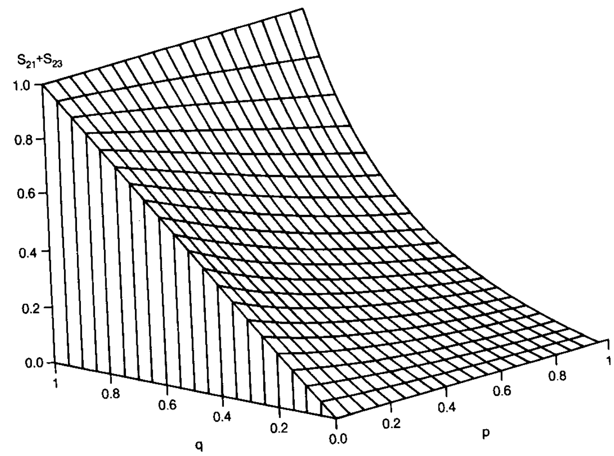



Fig. 5. The middle-to-side throughput in a three station system.

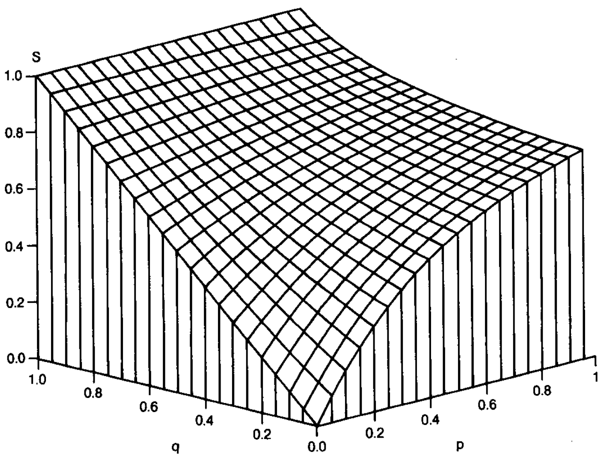



Fig. 6. The total throughput in a three station system.

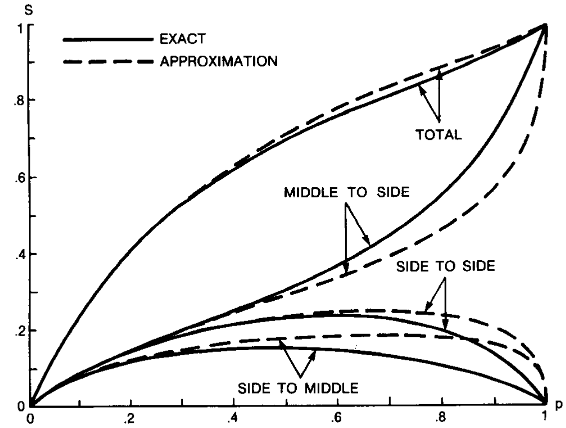



Fig. 7. The throughput in a fully symmetric three station system.

and destined to the other side node (the sum of $S_{13}$ and $S_{31}$) as a function of $p$ and $q$. Fig. 5 is a three-dimensional plot of the throughput originated at the middle node and destined to a side node (the sum of $S_{21}$ and $S_{23}$) as a function of $p$ and $q$. We omit plotting the throughput originated at the side nodes and destined to the middle node, since the shape of this curve is similar to the one given in Fig. 4 (however, the level of that curve is lower, e.g., for $p = 1$ and $q = 1$, its value is 1/4, while in Fig. 4 this value is 1/2). Lastly, Fig. 6 depicts the total throughput ($S$) in the system as a funciton of $p$ and $q$. A discussion of the system behavior, as observed in these figures, is given in Section IV-D.

### C. Polite System: An Approximation for an N Station System

Since the exact method described in the previous subsection may not be applied for systems with large $N$ (due to the exponential number of equations—$4^{N-1}$), we next propose an alternative approximation method.

Let the triple $(RS, k, t)$ [the triple $(LS, k, t)$] denote the event that during slot $t$ station $k$ does not hear a packet arriving from the right (left). Let the triple $(Q, k, t)$ denote the event that station $k$ is quiet (does not transmit) at slot $t$. To derive the system throughput, we first calculate the probability that the event $(LS, k, t)$ occurs. This event occurs *if and only if*, for every station $j$, such that $1 \le j \le k$, station $j$ does not transmit at time $t + j - k$. Thus,

$$\Pr[(LS, k, t)] = \Pr[(Q, k-1, t-1), (Q, k-2, t-2), \cdots, (Q, 1, t-k+1)]. \quad (4.1)$$

This can be calculated as

$$\Pr[(LS, k, t)] = \Pr[(Q, k-1, t-1) \mid (Q, k-2, t-2), \cdots, (Q, 1, t-k+1)] \cdot \Pr[(Q, k-2, t-2), \cdots, (Q, 1, t-k+1)]. \quad (4.2)$$

The conditional probability given above can be calculated as follows:

$$\Pr[(Q, k-1, t-1) \mid (Q, k-2, t-2), \cdots, (Q, 1, t-k+1)] = 1 - G_{k-1} \cdot \Pr[(RS, k-1, t-1) \mid (Q, k-2, t-2), \cdots, (Q, 1, t-k+1)]. \quad (4.3)$$

Now to calculate the expression

$$\Pr[(RS, k-1, t-1) \mid (Q, k-2, t-2), \cdots, (Q, 1, t-k+1)],$$

we make the following independence assumption for the system.

*1) Independence Assumption:* The event $(RS, k, t)$ is independent of the events $(Q, k-1, t-1), \cdots, (Q, 1, t-k+1)$.

This assumption means that the event that at time $t$ station $k$ hears a transmission arriving from the right is independent of the fact that stations $k-1, k-2, \cdots, 1$ are quiet at times $t-1, t-2, \cdots, t-k+1$, respectively. Obviously, this is not a true property of our system since these events are correlated to each other. However, it is easy to see that the dependency between these events is relatively weak, and thus we assume full independence. Note that this assumption will imply also that the event $(RS, k, t)$ is independent of the event $(LS, k, t)$.

We now assume that the system is at steady state and denote: $R_k \triangleq \Pr[(RS, k, t)]$, $L_k \triangleq \Pr[(LS, k, t)]$. Then from the independence assumption and from (4.2) and (4.3), we may conclude

$$L_k = (1 - R_{k-1} \cdot G_{k-1}) \cdot (1 - R_{k-2} \cdot G_{k-2}) \cdots (1 - R_1 \cdot G_1); \quad k = 2, 3, \cdots, N. \quad (4.4)$$

In a symmetric way, we calculate $R_k$

$$R_k = (1 - L_{k+1} \cdot G_{k+1}) \cdot (1 - L_{k+2} \cdot G_{k+2}) \cdots (1 - L_N \cdot G_N); \quad k = 1, 2, \cdots, N-1. \quad (4.5)$$

The values of $L_1$ and $R_N$ are obviously 1. Now (4.4) and (4.5) form a set of $2N - 2$ equations in $2N - 2$ variables, a set which can be solved by numerical methods.

Using the independence assumption and assuming steady state (see [10]), we may now calculate the system throughputs as a function of the parameters $R_k$ and $L_k$:

$$S_{jk} = G_j \cdot r_{jk} \cdot L_j \cdot R_j \cdot R_k; \quad j < k \quad (4.6a)$$

$$S_{jk} = G_j \cdot r_{jk} \cdot R_j \cdot L_j \cdot L_k; \quad j > k. \quad (4.6b)$$

From these equations and from (4.4) and (4.5), one can calculate the system throughput as a function of the transmission parameters.

Next we examine the quality of the approximation. We do so by computing the throughput for systems where the offered load is identical for all stations ($G_i = p$). For the three station system, the approximation results are compared to the exact results (derived in Section IV-B) and depicted in Fig. 7. For a five station system and a

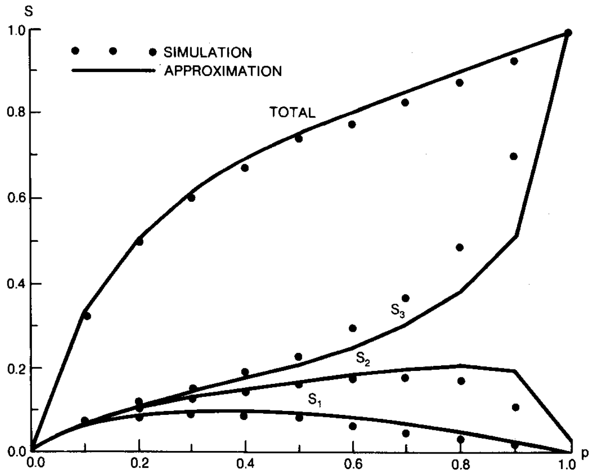



Fig. 8. The throughput in a fully symmetric five station system: simulation versus approximation.

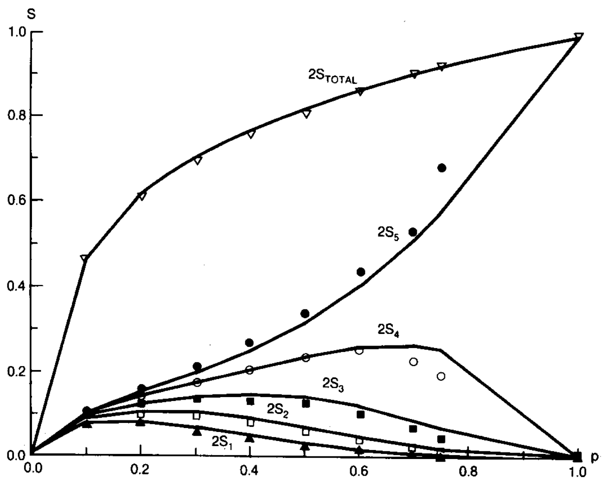



Fig. 9. The throughput in a fully symmetric ten station system: simulation versus approximation.

ten station system, the approximation results are compared to simulation. Fig. 8 depicts the throughput in the five station system—note that dots in this figure represent simulation results, and that the throughput $S_4$ and $S_5$ are not plotted (since $S_4 = S_2$ and $S_5 = S_1$). Fig. 9 depicts the throughput in a ten stations system; shaped dots represent simulation, and each curve represents the sum of the throughput for two symmetric stations (like 1 and 10).

For these comparisons we observe that the approximation predicts the individual station throughput quite accurately for low offered loads ($p < 0.6$) and not so accurately for higher offered loads. The reason is that at high loads the dependency between events increases and thus the independence assumption does not reflect the system behavior properly. Note, however, that the accuracy increases with the system size, so that for large systems the approximation may be quite accurate. In light of the discrepancies in predicting the individual station throughputs, the predictions for the total system throughput are surprisingly very accurate. It seems that the errors in predicting the individual station throughput, using the independence assumption, compensate for each other, yielding a very good approximation for the total throughput.

### D. Discussion of the Results

The analysis of Section IV reveals the important properties of the very fast bus system. These properties are discussed below.

From the analysis of the three station system (which is an exact analysis), we see how the system throughput is affected by the offered load of the individual stations. At the level of individual stations, we recognize that when a certain station increases its load, the throughput originated at this station will increase while the throughput originated at the other stations will decrease. This behavior is quite common for shared channel communication networks; for example, the slotted ALOHA system, or the nonpolite system described in Section IV-A above behave the same way [see equation (3.2)].

While at the individual station level the polite system behaves very much like other shared channel systems, the advantages of this system are revealed by examining the behavior of its total throughput. From Fig. 6 we observe that any increase in the offered load from either of the side stations or of the middle station causes an increment in the total throughput. The importance of this property is that the system is very stable: whenever the system load increases, the throughput also increases. This property is not very common in shared channel communication networks. For example, in slotted ALOHA, which is unstable (see, e.g., [8]), an increase in the offered load may cause the total throughput to decrease.

The importance of the stability property is that no special mechanism is required to create the system stability. In nonstable systems, like slotted ALOHA, it is required to control the offered load to prevent the system from getting into unstable states (states in which the system blocks itself); here these mechanisms are not required since the system controls itself in a natural way.

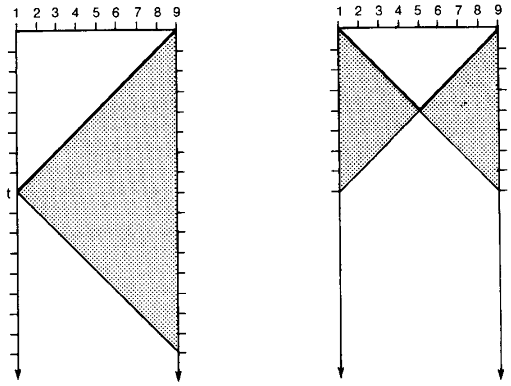
When the system is fully symmetric, its behavior is very similar. Figs. 7–9 show that at low load the throughput of every station increases, while at high load the middle stations become more and more dominant at the expense of the side stations which become more quiet. The total throughput, nonetheless, monotonically increases with the offered load.

An important property observed in all these systems (and systems of other sizes that we examined as well) is that the total throughput approaches 1 when the offered load of each station approaches 1. This is an important feature for stability in real systems in which buffering is used to queue messages and in which at very high loads all queues are likely to contain messages.

The explanation for this stability property is due to the fact that, unlike other shared channel systems, the stations in this system are not all alike. Rather, at every moment $t$, some stations get transmission priority over the others. More specifically, we may note that if station $i$ successfully transmitted at time $t - 1$, it has full transmission priority at time $t$ (due to the politeness), and thus, if it does transmit at this time, the transmission will be successful as well. This type of behavior leads the system to behave like an exhaustive service system in which a station who grabbed the channel will hold it for quite a while, while the other stations remain polite.

Note that this feature provides a possible explanation for stability, but does not prove it. The reason is that, in order to prove the property, the continuous-transmission periods must be weighted against contention periods, which can be lengthy as well and in which the throughput can drop to 0. The actual proof of the property, for the three station system, is given by the exact analysis provided above, which does take this weighting into consideration. For larger systems, strong evidence for the property is given by the simulation and approximation results provided, for example, in Figs. 7–9. Support for the presence of continuous transmission periods (as an explanation for stability) was provided by examining the transmission patterns observed in these simulations.

Another interesting property is the effect of station location on its "priority" within the system. From the results, it is evident that the central stations get higher priority than the side stations. In fact, the closer a station is to the center, the higher is its throughput. An intuitive explanation for this feature can be given by observing Fig. 10, in which we give an estimate for the likelihood of a station getting into a continuous-transmission mode. The heavy line in Fig. 10(a) identifies the set of points which needs to be silent to allow station 1 (a side station) to successfully transmit at point (1, $t$). The shaded area needs to be silent in order for the station to be




(a) (b)

Fig. 10. Requirements for continuous transmission: (a) side station; (b) central station.

undisturbed after $t$ (and thus not having to obey politeness) and get into a mode of "controlling the channel" and continuously transmitting. In Fig. 10(b), we depict the same regions for a central station. The size of the shaded area required for the side station is given by $N^2$, while the size of this area for the central station is given by $N^2/2$. This indicates that a central station will have much higher likelihood of taking control of the channel, and provides an explanation for its higher throughput.

While these properties have been observed with regard to the several systems we studied, it remains as an open question whether or not the properties hold for any size system; we conjecture that it is true for larger systems.

An interesting question is what would be the performance of a system which is implemented by two unidirectional buses and in which a station transmits a packet only on one bus to the downstream destination. We conjecture that the performance of this system will be better than or equal to that of the system analyzed in this paper (since it "generates less noise"). This superiority is, nonetheless, restricted to the stochastic model. If, instead, we consider a model of strict deterministic schedules, it is easy to see that both systems share the same bounds: the capacity of this system is 2 in a nonpolite environment (the proof of this property is embedded in the proof of Theorem 3) and 1 in a polite environment.

## V. Concluding Remarks

The main objective of this paper has been to investigate the projections from the previous literature regarding the degradation of efficiency in very fast bus networks. To pursue this investigation, we proposed a model which captures the dependency of events both on time *and* space. Using this model, we have unveiled the nonsymmetric structure of the bus architecture which had been previously obscured by the traditional assumption of uniform packet propagation time. The main predictions of our model are that the very fast channel is stable and that, in contrast to what has been predicted before, the channel efficiency does not severely degrade at very high transmission rates.

The need to model the time-space event dependency imposed technical complications on the model. As a result, we needed to simplify the model in other ways, such as assuming a uniform distribution of the stations over the bus. Further research is required, therefore, to provide *precise* throughput predictions for less simplified models. Nevertheless, we conjecture that, in general, the *principles* observed in this paper should hold for those models as well. Specifically, we conjecture that the property of system stability should hold when the stations are not evenly spaced over the bus; as a matter of fact, since the layout studied here (evenly spaced stations) is more symmetric than general layouts, it seems that the other layouts are likely to lead to even more stable systems. On the other hand, the property of reaching capacity of 2, using strict schedules, will not be enjoyed by all structures. For example, if one considers the evenly spaced layout, and removes several stations, one creates holes in the transmission pattern which degrade the throughput. In such cases, the capacity will likely lie between 1 and 2.

## Acknowledgment

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**Hanoch Levy** (S'83–M'86) for a photograph and biography see p. 1760 of this issue.